

# Roulette Physics

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## Abstract

The motion of a roulette ball on the rim of a tilted roulette wheel is studied. A mathematical model for the differential equation of motion and exit condition is derived, together with a solution procedure for obtaining values for the time and location when the ball leaves its circular track and begins its spiralling decent into the rotor. Special attention is given to real-time implementation possibilities.

KEYWORDS: ROULETTE PHYSICS, PHYSICAL PREDICTION, NEWTONIAN MECHANICS

## 1 Introduction

According to Edward O. Thorp, the key problem in trying to physically predict the outcome of a game of roulette is to "determine when and where on the stator the ball will leave the track, because then everything else except rotor velocity is a constant" (cf [1]). On tilted wheels, "the ball will not fall from a sector of the track on the high side; the effect is strong with a tilt of just  $0.2^\circ$ , which creates a forbidden zone of a quarter to a third of the wheel; the non-linear differential equation governing the balls motion on the track is the equation for a pendulum which at first swings completely around its pivot but is gradually slowed by air resistance" (cf [2]).

## 2 The Physical Model

### 2.1 Differential Equations of Motion

Applying Newtonian mechanics to the above leads to the following differential equations of motion in the time domain:

$$\dot{\theta} = \Omega, \quad \dot{\Omega} = -\alpha\Omega^2 + \beta - \gamma \sin \theta \quad (1)$$

with initial conditions at time  $t = 0$ :  $\theta(0) = 0$ ;  $\Omega(0) = \Omega_0$ ,

t[seconds]    time  
 $\theta$ [radians]    angular distance  
 $\Omega$ [rad/s]    angular velocity

and the dot denotes differentiation with respect to time (d/dt).

symbol	value	SI units	meaning
R	0.242	[m]	ball track radius of roulette wheel
S	$\pi r^2$	[m <sup>2</sup> ]	cross-section area of roulette ball
r	0.00874	[m]	radius of roulette ball
m	0.00374	[Kg]	mass of roulette ball
$C_w$	0.46-2.0	[1]	aerodynamic drag coefficient
$\rho$	1.2	[Kg/m <sup>3</sup> ]	density of air
g	9.807	[m/s <sup>2</sup> ]	gravitational acceleration
$\mu_0$	0.001	[1]	coefficient of rolling friction
$\mu_1$	0.014	[1]	coefficient of rolling/sliding friction
$\delta$	0.2574	[rad]	inclination angle of stator towards rotor
$\sin \delta$	0.2546		
$\cos \delta$	0.9670		stator inclination $\angle \delta = 14.75^\circ$
$\tan \delta$	0.2633		
$\varepsilon$	0.0035	[rad]	tilt $\angle \varepsilon = 0.2^\circ$
( $\sin \varepsilon \approx \varepsilon$ )			

Table 1: Values for physical and geometric constants

Here we have assumed that the roulette wheel is tilted at some dip angle  $\varepsilon$  to the horizontal and  $\theta$  is the angle, at time t, of the ball as measured from the center of the wheel to the ball, referenced from the radius which makes the lowest drop from the horizon.

The parameters  $\alpha, \beta$  and  $\gamma$  are given by:

$$\alpha = \frac{5}{7} \frac{S}{2m} C_w \rho R + (\mu_1 \cos \delta + \mu_0 \sin \delta) \quad (2)$$

$$\beta = \frac{5}{7} \frac{g}{R} [\mu_1 \sin \delta - \mu_0 \cos \delta] \quad (3)$$

$$\gamma = \frac{5}{7} \frac{g}{R} \sin \varepsilon \quad (4)$$

For our 60cm diameter roulette wheel the numerical values for the physical and geometric constants are given in Table 1, leading to a typical equation for the angular velocity of the ball:

$$\dot{\Omega} = -0.0225\Omega^2 + 0.075 - 0.1 \sin \theta \quad (5)$$

Equation (5) shows the magnitude of the forces involved. At higher velocities (e.g. 2rps corresponding to  $\Omega^2 \approx 160$ ) the first damping term dominates. At lower velocities (e.g. 0.5rps corresponding to  $\Omega^2 \approx 10$ ) the contributions of the other terms become significant.

Several remarks related to the physical model are in order ...

## 2.2 Air Resistance

The resistance of the air acts on the roulette ball causing its velocity to decrease over time. The law in force on the ball is determined by the Reynolds number, defined as

$$\text{Re} = \frac{\rho v \ell}{\eta},$$

where  $\rho$  and  $\eta$  are the density and viscosity of air,  $v$  is a characteristic velocity and  $\ell$  is a characteristic length. The drag force is usually expressed in the form

$$F = \frac{S}{2} C_w \rho v^2,$$

where the drag coefficient  $C_w$  is a dimensionless number which is a function of the Reynolds number. For values of Re of the order of 1 or less, the force is proportional to the velocity and  $C_w$  is proportional to  $1/\text{Re}$  (viscous damping). For values of Re of the order 1,000 to 100,000 the force is proportional to the square of the velocity and  $C_w$  is a constant (Newtonian damping). We have approximately  $\rho=1.2[\text{Kg}/\text{m}^3]$  and  $\eta=1.853 \cdot 10^{-5}[\text{Pa}\cdot\text{s}]$ . The diameter of the ball  $\ell$  is  $\approx 0.02[\text{m}]$  and the characteristic velocity is  $v=1.52[\text{m}/\text{s}]$  ( $=1[\text{rps}]=2\pi[\text{rad}/\text{s}]$ ). The maximum Reynolds number based on the diameter of the ball is 1946 so that the quadratic force law and *Newtonian damping* should apply.

The aerodynamic drag coefficient for a sphere falling through the air is approximately 0.46. Because of the geometry of the ball rolling on a circular track inside the roulette wheel we expect the actual aerodynamic drag coefficient to be higher than 0.46, maybe up to 2.0, approaching the aerodynamic drag coefficient of a hollow semi-cylinder facing stream.

However, air resistance alone cannot account for the velocity decay of the ball. If we assume for the moment that we have no rolling friction ( $\mu_0, \mu_1=0$ ), then the deceleration parameter  $\alpha=(5/7)(S/2m)C_w\rho R$  for the ball would be in the range between 0.0035 (for  $C_w \approx 0.5$ , i.e. laminar air flow) and 0.0135 (for  $C_w \approx 2$ , i.e. turbulent air flow due to roulette wheel geometry). But we know from observation of our typical wheel/ball configuration that the ball velocity decays from approximately 0.83 revolutions per second (rps) to 0.45 rps in about 10 seconds and this would require the parameter  $\alpha$  to be in the range between 0.018 and 0.02.

Consequently we need to consider the effects due to ...

### 2.3 Surface Friction

In a private communication Edward Thorp pointed out the following observations concerning the complex orbital motion of the spinning roulette ball:

"I believe that when we [Claude Shannon and I] did tests with our standard casino wheel we noted that there were certain orbital complexities arising from the interchange of energy between orbital angular momentum and spin angular momentum, as well as frictional effects such as the change of mode from sliding friction when the ball is initially launched to a kind of rolling friction thereafter. There is also the behaviour of the axis of rotation of the ball: If the ball rolls on the track, the axis of rotation precesses around a vertical direction and so there must be some frictional 'slippage', perhaps 'noisy' to accommodate this. Whether any of these things are significant enough to be accounted for, I do not know, but I suspect they matter at higher velocities, say more than 5-6 revolutions before the ball leaves the track."

That said, it is clear that the motion of the roulette ball on the rim of the wheel is divided into two phases: the first phase in which the ball slides/rolls and during which (with the help of sticky friction) slipping and abnormal spin angular momentum is converted to orbital energy; and then the second phase where slipping has ceased and the ball is performing a more or less regular rolling motion and where the points of contact between the ball and the rim of the wheel trace out a 3-dimensional roulette curve.

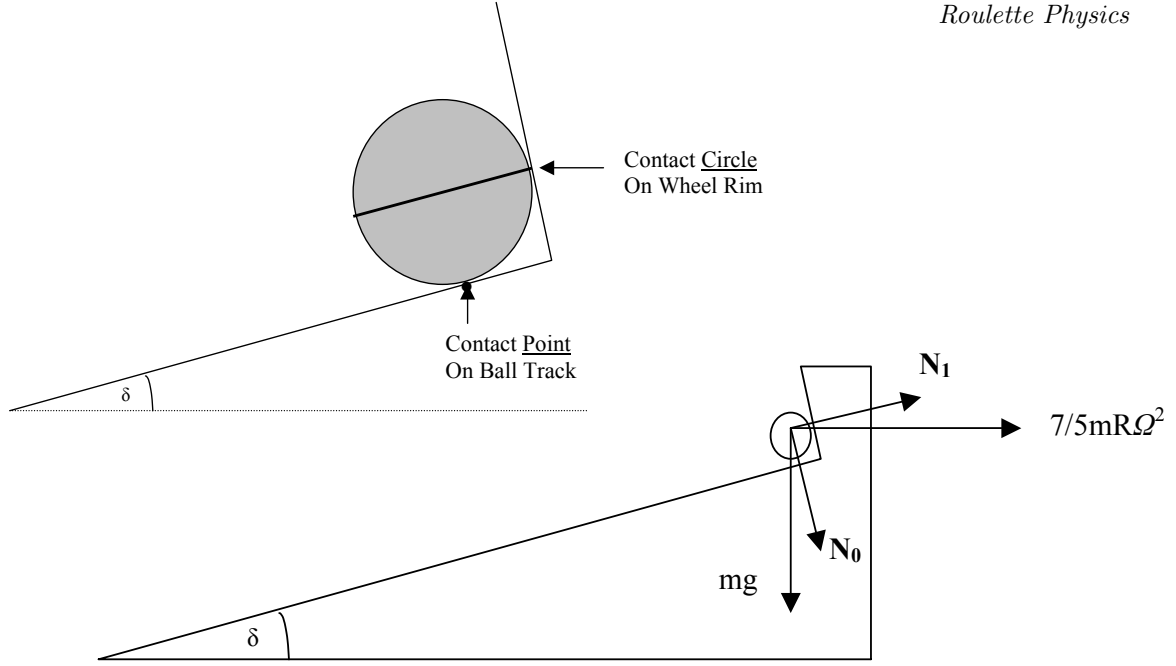


Figure 1: *Roulette Wheel side-view showing normal forces and contact circles*

Our physical model described by equations (1) is only valid *after* the ball has connected to the **roulette** and *before* it disconnects from the **roulette** and leaves the track.

The force resulting from rolling friction is usually modelled as

$$\mathbf{F} = \mu \mathbf{N},$$

where  $\mu$  is the coefficient of rolling friction and  $\mathbf{N}$  is the force acting in the normal direction. Consequently, we have included in our physical model two friction force terms according to the two normal forces on the ball:

$$\begin{aligned} \mathbf{F}_1 &= \mu_1 \mathbf{N}_1 = \mu_1 m \left[ \frac{7}{5} R \Omega^2 \cos \delta - g(\sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \cos \theta) \right] \\ \mathbf{F}_0 &= \mu_0 \mathbf{N}_0 = \mu_0 m \left[ \frac{7}{5} R \Omega^2 \sin \delta + g(\cos \delta \cos \varepsilon - \sin \delta \sin \varepsilon \cos \theta) \right] \end{aligned}$$

The first term corresponds to the friction force when the ball is pressing against the rim perpendicular to the circular ball track and the second term corresponds to the friction force of the ball rolling on its inclined track. The coefficient  $\mu_1$  is an order of magnitude larger than  $\mu_0$  because of the arc of contact between the rim and ball which forces the ball onto its circular track causing micro-slippage. The small value of  $\mu_0$  (relative to  $\mu_1$ ) reflects the fact that there is only a small 'drilling' friction between the track and the ball.

For small  $\varepsilon$  we can set  $\cos \varepsilon = 1$  and ignore the terms with coefficients  $\mu_1 \cdot \sin \varepsilon$  and  $\mu_0 \cdot \sin \varepsilon$ . After rearranging, we get for the friction contributions:

$$\begin{aligned} [\mu_0 \sin \delta + \mu_1 \cos \delta] \frac{7}{5} m R \Omega^2 & \quad (\text{friction component proportional to centrifugal force}) \\ [\mu_0 \cos \delta - \mu_1 \sin \delta] m g & \quad (\text{friction component proportional to gravitational force}) \end{aligned}$$

As  $\mu_0$  is an order of magnitude smaller than  $\mu_1$ , the friction component corresponding to the gravitational force has negative(!) sign; hence the positive(!) sign of  $\beta$  in equation (1).

The factor of  $5/7$  is included in the model to account for the inertia due to the rotational motion of the roulette ball which is rolling without slipping.

## 2.4 Summary of the Physical Model

We are pleased to find that, notwithstanding the complex dynamics of the system and the different nature of the forces involved, all the contributing terms can be condensed into the compact model equations given in (1).

Of course, we do not need to and cannot know all of the physical parameters mentioned above precisely. Estimates of them are however helpful in determining and checking the validity of the value ranges of the aggregate model parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  (which in turn must be calculated precisely!).

We will show that the physical model describes quite accurately what is actually happening (if the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  are correctly identified from experimental data) and that equations (1) are useful for making predictions.

## 3 The Case without Tilt

We will first study the solution of the simplified model where we assume that the wheel is perfectly level ( $\gamma=0$ ). In this case, equations (1) reduce to:

$$\dot{\theta} = \Omega, \quad \dot{\Omega} = -\alpha\Omega^2 + \beta; \quad \alpha, \beta > 0 \quad (6)$$

By setting  $a = \alpha$  and  $b^2 = \beta/\alpha$ , the second equation in (6) becomes

$$\dot{\Omega} = -a(\Omega^2 - b^2) \quad (7)$$

which is directly integrated to give:

$$\Omega(t) = -b \coth(c_0 - ab \cdot t) \quad (8)$$

The initial condition  $\Omega(0) = \Omega_0$  defines the integration constant  $c_0$  to:

$$c_0 = -\operatorname{arcoth}\left(\frac{\Omega_0}{b}\right) \quad (9)$$

For  $\Omega_0 > b$  we have  $c_0 < 0$  and  $\Omega(t) > b > 0$  for all  $t > 0$ .

A further integration gives:

$$\theta(t) = \frac{1}{a} \ln \left[ \frac{\sinh(c_0 - ab \cdot t)}{\sinh c_0} \right] \quad (10)$$

with  $\theta(0) = 0$ .

The above equation can be solved for  $t$ :

$$t(\theta) = \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a\theta}) \right] \quad (11)$$

with  $t(0) = 0$ .

By inserting (11) into (8), using identities of the hyperbolic transcendental functions and carefully observing  $\pm$  signs, we obtain  $\Omega$  as a function of  $\theta$ :

$$\Omega(\theta) = \sqrt{c_1 e^{-2a\theta} + b^2} \quad (12)$$

with the constant  $c_1$  defined by

$$c_1 = \Omega_0^2 - b^2 = b^2(\coth^2 c_0 - 1) \quad (13)$$

with  $\Omega_0 = \Omega(\theta = 0)$ .

If we know the time  $T_0$  for one revolution, we can determine  $c_0$  and  $c_1$ :

$$x = \frac{\Omega_0}{b} = \frac{e^{a \cdot 2\pi} - \cosh(ab \cdot T_0)}{\sinh(ab \cdot T_0)} \quad (14)$$

$$c_0 = -\operatorname{arccoth}(x) \quad (15)$$

$$c_1 = b^2(x^2 - 1) \quad (16)$$

The ball will come off its circular track and begin its spiralling decent into the rotor when the pull of gravity overcomes the centrifugal acceleration:

$$\Omega_f^2 = \frac{5}{7} \frac{g}{R} \tan \delta \approx 7.62 [\text{rad}^2/\text{s}^2] \quad (17)$$

Equating  $\Omega^2(\theta)$  with  $\Omega_f^2$  gives the fall position  $\theta_f$ :

$$\theta_f = \frac{1}{2a} \ln \left[ \frac{c_1}{\Omega_f^2 - b^2} \right] \quad (18)$$

We then obtain the fall time  $t_f$  by evaluating (11) at  $\theta_f$ :

$$t_f = t(\theta_f) \quad (19)$$

On a perfectly level wheel there is no 'forbidden zone' where the ball cannot exit and for different initial conditions  $\Omega_0$  all exit angles  $\theta_f \in [0, k \cdot 2\pi]$  are possible.

## 4 The Case with Tilt

### 4.1 Phase-Plane Solution

For  $\gamma > 0$  there exists no closed-form solution of (1) in the time domain. However, in the *phase-plane* the trajectories  $\theta(t), \Omega(t)$  satisfy the equation

$$\left(\frac{d\Omega}{dt}\right) / \left(\frac{d\theta}{dt}\right) = \frac{d\Omega}{d\theta} = \frac{-\alpha\Omega^2 + \beta - \gamma \sin \theta}{\Omega} \quad (20)$$

So, the 'curve'  $\Omega(\theta)$  corresponding to the differential equations of motion (1) is defined by the differential equation

$$(\Omega^2)' = -2\alpha\Omega^2 + 2\beta - 2\gamma \sin \theta \quad (21)$$

where the dash (') denotes differentiation with respect to angular distance (d/d $\theta$ ).

Substituting  $\Gamma(\theta) = \Omega^2(\theta)$  gives

$$\Gamma' = -2\alpha\Gamma + 2\beta - 2\gamma \sin \theta \quad (22)$$

It is somewhat surprising that a closed-form solution for the linear, first-order differential equation (22) exists. Here it is:

$$\Gamma(\theta) = c_1 e^{-2\alpha\theta} + \frac{\beta}{\alpha} + \frac{2\gamma}{4\alpha^2 + 1} (\cos \theta - 2\alpha \sin \theta) \quad (23)$$

Thus, on tilted wheels the angular velocity  $\Omega$  as a function of angular distance  $\theta$  is given by:

$$\Omega(\theta) = \sqrt{c_1 e^{-2a\theta} + b^2 + \eta(\cos \theta - 2a \sin \theta)} \quad (24)$$

with  $a = \alpha$ ,  $b^2 = \beta/\alpha$ ,  $\eta = 2\gamma/(4\alpha^2 + 1)$  and the constant  $c_1$  defined by the initial condition  $\Omega(0) = \Omega_0$ :

$$c_1 = \Omega_0^2 - b^2 - \eta \quad (25)$$

The inclusion of  $\eta$  in the above equation is a consequence of our chosen coordinate system, with  $\theta = 0$  at the low point of the wheel.

In practice it is not always feasible to fix the initial conditions to  $\theta(0) = 0$  at time  $t = 0$ . If we have instead  $\theta(t = 0) = \varphi$  and  $\Omega(\varphi) = \Omega_0$ , the integration constant  $c_1$  becomes:

$$c_1 = [\Omega_0^2 - b^2 - \eta(\cos \varphi - 2a \sin \varphi)] e^{2a\varphi} \quad (26)$$

When  $c_1$  is known, the solution curve  $\Omega(\theta)$  is completely defined through (24).

## 4.2 Exit Condition

The exit condition on a tilted wheel is obtained by observing that the ball will leave the track when the pull of gravity overcomes the centrifugal acceleration:

$$\Omega_f^2 = \frac{5}{7} \frac{g}{R} [\tan \delta \cos \varepsilon - \sin \varepsilon \cos \theta_f] \quad (27)$$

or, taking typical values<sup>1</sup>:

$$\Omega_f^2 = 7.62 - 0.1 \cos \theta_f \quad (28)$$

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<sup>1</sup>A derivation of formula (27) for the exit condition including tilt is given in section 6

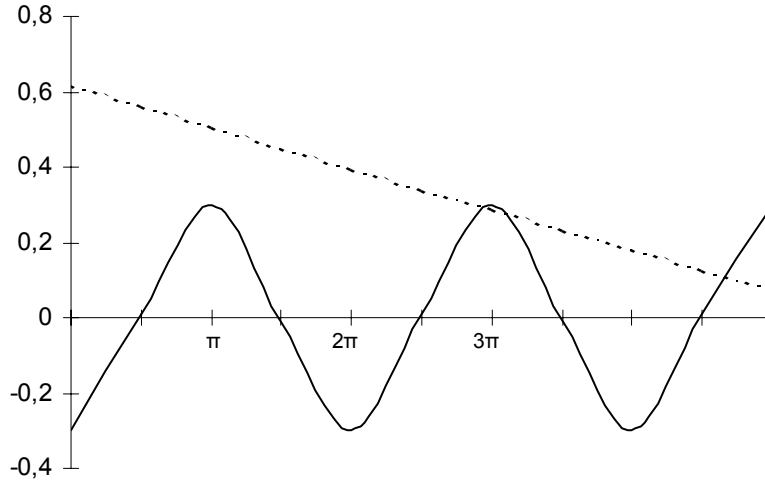


Figure 2: *The Fall Position,  $\theta_f$ , is the first point of intersection between the two curves*

### 4.3 Exit Location

The exit location,  $\theta_f$ , can then be calculated by equating  $\Omega^2(\theta) = \Omega_f^2$ :

$$c_1 e^{-2a\theta} + \eta \left[ \left(1 + \frac{1}{2}(4a^2 + 1)\right) \cos \theta - 2a \sin \theta \right] + b^2 - \frac{5}{7} \frac{g}{R} \tan \delta \cos \varepsilon = 0 \quad (29)$$

or, for typical values:

$$c_1 e^{-0.045\theta} + \eta [1.501 \cos \theta - 0.045 \sin \theta] + 3.33 - 7.62 = 0 \quad (30)$$

or (with  $\eta = 0.2$  corresponding to  $\varepsilon = 0.2^\circ$ ):

$$c_1 e^{-0.045\theta} + 0.3 \cos \theta - 0.009 \sin \theta + 4.29 = 0 \quad (31)$$

In equation (30) the actual dependence of the solution on  $\eta$  and thus  $\varepsilon$  is retained. We must solve for the smallest positive root to obtain the fall position,  $\theta_f$ . Beyond  $\theta_f$ , the ball "disconnects" from the solution curve (24) and begins its spiralling path into the rotor.

#### 4.3.1 Graphical Solution

Figure 2 shows the graphical solution for equation (31). The dotted line corresponds to the function  $g_1(\theta) = c_1 e^{-0.045\theta} - 4.29$  and the solid line corresponds to the function  $g_2(\theta) = -\eta [1.501 \cos \theta - 0.045 \sin \theta]$ , with the abscissa of the first point of intersection of the two curves defining the desired root  $\theta_f$ .

As can be seen, the tilt angle  $\varepsilon = 0.2^\circ$  is sufficiently large to "shade" an angular region where a solution of (31) is not possible thus creating "a forbidden zone" on the wheel where the ball cannot exit. The forbidden zone is approximately located between the angles  $\pi$  and  $3\pi/2$ , which corresponds to a section on the high side of the wheel.

The existence of a forbidden zone of a quarter to a third of the wheel for a tilt angle of  $0.2^\circ$  causes the fall positions of the ball to differ greatly from those of a wheel without tilt.



### 4.3.2 Numerical Solution

As can also be seen on the above graph is that there may be multiple intersection points of  $g_1(\theta)$  and  $g_2(\theta)$  corresponding to multiple roots of the transcendental, non-linear equation  $g(\theta) = 0$ .

For a robust numerical root-finding procedure we first need to make a good initial guess as to where the smallest positive root is located, i.e. we need to "trap" the root between two bracketing values, and then hunt it down like a rabbit. It is illuminating to make sketches showing the geometric distribution of roots of  $g(\theta)$  corresponding to different initial conditions and different parameter settings. This also gives clues as how to organize the hunt for the desired root.

Because the derivative of  $g(\theta)$  is easy to calculate, *Newton's Method* (which generates successive iterates by approximating the curve  $g(\theta)$  by its tangent line) is the method of choice. The procedure converges very rapidly to the solution provided the initial guess is a good one. An efficient algorithm for finding the bracketed root of a differentiable function is described in (cf [3]). The described routine **rtsafe** uses a combination of *Newton's Method* and *bisection* to "home in" on the solution between the two bracketing values.

For an initial value  $\Omega_0 = 4.5$  (7.5), corresponding to  $\approx 0.7$  (1.2) rev/sec, the "distance-to-fall",  $\theta_f$ , is 31.75 (57.10) radians, or 5.05 (9.09) revolutions. A solution accurate to five digits was obtained by the routine **rtsafe** in five iterations, confirming that the method is quite adequate for calculations in real-time.

## 4.4 Time-of-Fall

After having determined the angular distance  $\theta_f$  that the ball travels after the initial measurement of  $\Omega_0$  at  $\theta = 0$ , the time-of-fall  $t_f$  can be calculated from equation (24) using the relationship  $\frac{d\theta}{dt} = \Omega$  or  $dt = \frac{1}{\Omega}d\theta$ . The resulting integral

$$t_f = \int_0^{\theta_f} \frac{d\theta}{\sqrt{c_1 e^{-2a\theta} + b^2 + \eta(\cos \theta - 2a \sin \theta)}} \quad (32)$$

which needs to be evaluated on the interval  $[0, \theta_f]$  to obtain  $t_f$ , is one of the ugliest one can possibly come across.

Brute force numerical integration by subroutine SIMP of (cf [4]) using our typical parameter values gives  $t_f = 9.37$  (13.79) seconds (for a fall position of  $\theta_f = 31.75$  (57.10) radians).

A more subtle approach, suitable for real-time calculations, is to split the value  $\theta_f$  into:

$$\theta_f = k \cdot 2\pi + \Delta\theta_f, \quad k \text{ an integer}, \quad (33)$$

and evaluate the integral (32) on the two sub-intervals  $[0, k \cdot 2\pi]$  and  $[k \cdot 2\pi, \theta_f]$  to get

$$t_f = t(k \cdot 2\pi) + \Delta t_f \quad (34)$$

Looking at the first sub-interval  $[0, k \cdot 2\pi]$ , it can be shown that for 'complete revolutions' (32) reduces to the formula

$$t(k \cdot 2\pi) = \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a \cdot k \cdot 2\pi}) \right] \quad (35)$$

with  $c_0 = -\operatorname{arsinh}\left(\frac{b}{\sqrt{c_1}}\right)^2$

*Proof:*

It suffices to integrate (32) on the interval  $[0, 2\pi]$  to show the point. From (24) we get

$$\Omega^2(\theta) - (c_1 e^{-2a\theta} + b^2) = \eta(\cos \theta - 2a \sin \theta)$$

Integrating both sides from 0 to  $2\pi$  gives

$$\int_0^{2\pi} \left[ \Omega^2(\theta) - (c_1 e^{-2a\theta} + b^2) \right] d\theta = 0$$

or

$$\int_0^{2\pi} \left[ (\Omega(\theta) + \sqrt{c_1 e^{-2a\theta} + b^2}) \cdot (\Omega(\theta) - \sqrt{c_1 e^{-2a\theta} + b^2}) \right] d\theta = 0$$

and thus, because  $\Omega(\theta)$  and  $\sqrt{c_1 e^{-2a\theta} + b^2}$  are positive functions for  $\theta > 0$ :

$$\int_0^{2\pi} \left[ \Omega(\theta) - \sqrt{c_1 e^{-2a\theta} + b^2} \right] d\theta = 0$$

and consequently, since  $\sqrt{c_1 e^{-2a\theta} + b^2}$  is monotonically decreasing for  $\theta > 0$ :

$$\int_0^{2\pi} \frac{d\theta}{\Omega(\theta)} = \int_0^{2\pi} \frac{d\theta}{\sqrt{c_1 e^{-2a\theta} + b^2}}$$

So we have:

$$t(2\pi) = \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a \cdot 2\pi}) \right]$$

If we know the time  $T_0$  for one revolution, we can determine  $c_0$  and  $c_1$ :

$$x = \frac{e^{a \cdot 2\pi} - \cosh(ab \cdot T_0)}{\sinh(ab \cdot T_0)}$$

$$c_0 = -\operatorname{arcoth}(x)$$

$$c_1 = b^2(x^2 - 1)$$

Thus, for complete revolutions, the elapsed time  $t(k \cdot 2\pi)$  is the same for a tilted roulette wheel and one without tilt (with  $c_1 = \Omega_0^2 - b^2 - \eta$ ). This is in agreement with the observation that on a tilted roulette wheel the ball alternately gains momentum or loses momentum as it passes the high point or the low point of the wheel, respectively. And these two effects cancel for each complete revolution.

No short-cut exists, however, for integrating (32) on the second sub-interval  $[k \cdot 2\pi, \theta_f]$ . For small tilt angles  $\varepsilon$ , i.e. small values of  $\eta$  we suggest the following approach to calculate  $\Delta t_f$ .

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<sup>2</sup> $\operatorname{arsinh}(x) = \ln(x + \sqrt{x^2 + 1})$

A Taylor-expansion of the integrand  $f(\theta; \eta)$  gives:

$$f(\theta; \eta) = f(\theta; 0) + \eta \cdot \frac{\partial f(\theta; 0)}{\partial \eta} + \frac{\eta^2}{2} \cdot \frac{\partial^2 f(\theta; \zeta)}{\partial \eta^2}; \quad \zeta \in [0, \eta]$$

leading to the following representation for  $\Delta t_f$ :

$$\Delta t_f = \int_{k \cdot 2\pi}^{\theta_f} \left[ \frac{1}{\sqrt{c_1 e^{-2a\theta} + b^2}} - \frac{\eta (\cos \theta - 2a \sin \theta)}{2 \sqrt{(c_1 e^{-2a\theta} + b^2)^3}} + \frac{\eta^2}{2} \frac{\frac{3}{4} (\cos \theta - 2a \sin \theta)^2}{\sqrt{(c_1 e^{-2a\theta} + b^2 + \zeta (\cos \theta - 2a \sin \theta))^5}} \right] d\theta$$

The integral corresponding to the first term can be folded into equation (35). For the integral corresponding to the last term we get:

$$\begin{aligned} \frac{\eta^2}{2} \frac{3}{4} \int_{k \cdot 2\pi}^{\theta_f} \left[ \frac{(\cos \theta - 2a \sin \theta)^2}{\sqrt{(c_1 e^{-2a\theta} + b^2 + \zeta (\cos \theta - 2a \sin \theta))^5}} \right] d\theta &\leq \frac{\eta^2}{2} \frac{3}{4} 2\pi \cdot \max \frac{(1 + 2a)^2}{\sqrt{(c_1 e^{-2a\theta} + b^2 - \eta)^5}} \\ &\leq \frac{\eta^2}{2} \frac{3}{4} 2\pi \cdot \frac{(1 + 2a)^2}{\sqrt{(\bar{\Omega}_f^2 - \eta)^5}} \approx \frac{\eta^2}{2} \frac{3}{4} 2\pi \frac{1.092}{\sqrt{7^5}} \approx 0.02\eta^2 \approx 0.0008 \quad (\eta = 0.2) \end{aligned}$$

Thus, for our typical parameters and an average value of the square of the fall velocity of  $\bar{\Omega}_f^2 \approx 7.62$ , the contribution of the last term to  $\Delta t_f$  (and also to  $t_f$ ) is below 1 millisecond and can be neglected. Even for a large tilt of  $\eta = 0.6$ , the error in neglecting the second term of the Taylor-expansion would be below 0.008 seconds. A value of  $\eta = 0.6$  corresponds to a tilt angle of  $\varepsilon = 0.6^\circ$  or 6mm on our wheel of 600mm diameter. Higher tilt angles than  $\varepsilon = 0.6^\circ$  are very unlikely as they would extremely bias the fall position of the ball.

Coming back to the second term (the 1st term in the Taylor-expansion), an integration by parts yields:

$$\int_{k \cdot 2\pi}^{\theta_f} \left[ \frac{\cos \theta - 2a \sin \theta}{\sqrt{(c_1 e^{-2a\theta} + b^2)^3}} \right] d\theta = \left[ \frac{\sin \theta + 2a \cos \theta}{\sqrt{(c_1 e^{-2a\theta} + b^2)^3}} \right]_{k \cdot 2\pi}^{\theta_f} + 3a \int_{k \cdot 2\pi}^{\theta_f} \frac{(\sin \theta + 2a \cos \theta) c_1 e^{-2a\theta}}{\sqrt{(c_1 e^{-2a\theta} + b^2)^5}} d\theta$$

The absolute value of the second term on the right-hand side of the equal sign is bounded by:

$$\left| 3a \int_{k \cdot 2\pi}^{\theta_f} \frac{(\sin \theta + 2a \cos \theta) c_1 e^{-2a\theta}}{\sqrt{(c_1 e^{-2a\theta} + b^2)^5}} d\theta \right| \leq 3a \int_{k \cdot 2\pi}^{\theta_f} \frac{1 + 2a}{\sqrt{(c_1 e^{-2a\theta} + b^2)^3}} d\theta \leq \frac{0.44}{\sqrt{7^3}} \approx 0.024$$

So, dropping this term will lead to an error of  $\frac{\eta}{2} \cdot 0.024 = 0.0024$  seconds for  $\eta = 0.2$  (or 0.0072 seconds for  $\eta = 0.6$ ) in determining  $\Delta t_f$ .

Lastly, we have:

$$\left[ \frac{\sin \theta + 2a \cos \theta}{\sqrt{(c_1 e^{-2a\theta} + b^2)^3}} \right]_{k \cdot 2\pi}^{\theta_f} = \frac{\sin \theta_f + 2a \cos \theta_f}{\sqrt{(c_1 e^{-2a\theta_f} + b^2)^3}} - \frac{2a}{\sqrt{(c_1 e^{-2a \cdot k \cdot 2\pi} + b^2)^3}}$$

We can again drop the second term on the right-hand side which is  $<0.0024$  for our typical parameter values, retaining only the first term as a first-order correction for  $\Delta t_f$ .

Summing up we have as an approximation to (32):

$$t_f = \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a\theta_f}) \right] - \frac{\eta}{2} \frac{\sin \theta_f + 2a \cos \theta_f}{\sqrt{(c_1 e^{-2a\theta_f} + b^2)^3}} \quad (36)$$

which can easily be computed in real-time and where the worst-case error is  $\approx 0.0056$  seconds for the case  $\eta = 0.2$  and  $\approx 0.018$  seconds for the case  $\eta = 0.6$ .

If we assume that the rotor velocity is approximately 0.33 r.p.s, an error of 0.0056(0.018) seconds in calculating  $t_f$  would translate to an error of 0.07(0.22) pockets on the rotor.

#### 4.5 The Roulette Equations

We conclude this section by determining adjustments to the formulas for calculating the fall-position and the fall-time due to any change in the local coordinate system.

Up to now it was tacitly assumed that the point  $\theta = 0$  was the low point of the tilted wheel and at the same time was our "observation" point, i.e. the origin of our coordinate system. In practice this choice of origin is usually impossible as the measurement reference is usually one of the (vertical) metal deflectors on the stator which rarely coincides with the low point of the wheel. Also, some of the deflectors are inconvenient to be used as reference points as the corresponding positions on the wheel could be blocked by e.g. the croupier or other persons.

Let us suppose that our vantage point is located at some angle  $\varphi$  from the low point of the wheel. A calculation shows that if we choose  $\varphi$  as the new origin of our coordinate system, our formulas for determining  $\theta_f$  and  $t_f$  become:

$$c_1 e^{-2a\theta_f} + \eta \left[ \left(1 + \frac{1}{2}(4a^2 + 1)\right) \cos(\theta_f + \varphi) - 2a \sin(\theta_f + \varphi) \right] + b^2 - \frac{5}{7} \frac{g}{R} \tan \delta \cos \varepsilon = 0 \quad (37)$$

$$t_f = \int_0^{\theta_f} \frac{d\theta}{\sqrt{c_1 e^{-2a\theta} + b^2 + \eta(\cos(\theta + \varphi) - 2a \sin(\theta + \varphi))}} \quad (38)$$

or (approximately) :

$$t_f = \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a\theta_f}) \right] - \frac{\eta}{2} \frac{\sin(\theta_f + \varphi) + 2a \cos(\theta_f + \varphi)}{\sqrt{(c_1 e^{-2a\theta_f} + b^2)^3}} \quad (39)$$

The above formulas are the **roulette equations** for the fall-position and fall-time of the ball which are valid on wheels with or without tilt.

### 5 Parameter Estimation

In order to be able to make predictions from measurements of the initial angular velocity, the parameters  $a, b, \eta, \varphi$  and  $\bar{\Omega}_f^2 = \frac{5}{7} \frac{g}{R} \tan \delta \cos \varepsilon$  which are characteristic for a specific ball and wheel need to be identified from observed data.

If we make timing measurements  $t_k$  for complete revolutions of the ball (i.e. when the ball repeatedly passes a fixed reference point on the stator such as a vertical deflector), the ball deceleration parameters  $a, b$  can be identified by using equation (35) and minimizing

$$S(a, b, c_0) = \sum_{k=0}^n \left\{ t_k - \frac{1}{ab} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a \cdot k \cdot 2\pi}) \right] \right\}^2 \quad (40)$$

In the above equation the parameters  $a, b$  and  $c_0$  appear nonlinearly. Here, the method of Levenberg-Marquardt for nonlinear least-squares estimation as described in (cf [3]) is appropriate. If we clock the ball during the last 4 to 6 revolutions before it comes off the track we obtain 5 to 7 measurement points to determine the parameters. Repeating this process for several spins and taking arithmetic means of the identified parameters of each spin will reduce the influence of measurement errors in clocking the ball.

After the shape of the deceleration curve for the ball has been identified in the above way the parameter estimates can then be further refined by freezing the value for  $\beta = ab^2$  and minimizing<sup>3</sup>

$$S(a) = \sum_{k=0}^n \left\{ t_k - \frac{1}{\sqrt{a}\sqrt{\beta}} \left[ c_0 - \operatorname{arsinh}(\sinh c_0 \cdot e^{a \cdot k \cdot 2\pi}) \right] \right\}^2 \quad (41)$$

with

$$\begin{aligned} c_0 &= -\operatorname{arcoth}(x) \\ x &= \frac{e^{a \cdot 2\pi} - \cosh(ab \cdot T_0)}{\sinh(ab \cdot T_0)} \\ T_0 &= t_1 - t_0 \end{aligned}$$

The direct minimization of  $S(a)$  can be achieved through a Golden-Section-Search as described in (cf [3]) or by the method of trisection (cf [6]).

Once  $a, b$  are known, the remaining parameters  $\eta, \varphi$  and  $\bar{\Omega}_f^2 = \frac{5}{7} \frac{g}{R} \tan \delta \cos \varepsilon$  can be determined by minimizing

$$S(\eta, \varphi, \bar{\Omega}_f^2) = \sum_{k=1}^n \left| c_1 e^{-2a\theta_{f_k}} + \eta \left[ \left(1 + \frac{1}{2}(4a^2 + 1)\right) \cos(\theta_{f_k} + \varphi) - 2a \sin(\theta_{f_k} + \varphi) \right] + b^2 - \bar{\Omega}_f^2 \right| \quad (42)$$

with

$$\begin{aligned} c_1 &= b^2(x^2 - 1) \\ x &= \frac{e^{a \cdot 2\pi} - \cosh(ab \cdot T_0)}{\sinh(ab \cdot T_0)} \end{aligned}$$

Here we have only one nonlinear parameter  $\varphi$ . For successive spins, measured values for  $T_0$  and  $\theta_f$  are stored in a table and then (42) is directly minimized by trisection of the interval  $[0, 2\pi]$  which must contain the value of  $\varphi$ . For any fixed  $\varphi$ , equation (42) becomes

$$S(\eta, \bar{\Omega}_f^2) = \sum_{k=1}^n |y_k + \eta x_k + (b^2 - \bar{\Omega}_f^2)| \quad (43)$$

which can be solved by the method of robust linear estimation as described in (cf [3]).

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<sup>3</sup>this refinement is especially well suited to the way predictions are made during actual play

Minimizing the absolute value of  $S(\eta, \varphi, \bar{\Omega}_f^2)$  (instead of using least-squares minimization) is less sensitive to small errors in the measurement of  $T_0$  (which can lead to large errors in  $\theta_f$  due to the existence of the forbidden zone on tilted wheels)<sup>4</sup>.

Measuring  $\theta_f$  is done in the following way: mentally divide the wheel into  $n$  sectors; from a fixed reference point count the number of complete revolutions  $j$  (after the the initial clock pulse) before the ball leaves the rim of the wheel; determine the number  $i$  of sectors that the ball travels in the final round; calculate  $\theta_f = (j + \frac{i}{n}) \cdot 2\pi$ .

With training it is possible to identify by eyesight<sup>5</sup> the fall position of the ball to within a standard error of plus/minus two to three pockets and to reduce the timing error in determining  $T_0$  to approximately one hundredth of a second. The total standard error in predicting the fall position of the ball is then below four pockets.

## 6 Ball Rolling in a Circular Cone

When the ball leaves the rim of the roulette wheel it spirals down towards the rotor until it (usually) hits one of the metal deflectors before hitting the rotor and finally coming to rest in one of the pockets. This section deals with that part of the motion of the ball from the exit location up to the collision with a metal deflector. In low-profile roulette wheels, as they are common in the casinos today, the spiral arc is quite long and its length differs according to the exit location of the ball, the level of tilt and the geometric arrangement of the deflectors. From rigid body physics it is known that the kinetic energy of a sphere rolling on a smooth surface without slipping is given by

$$T = \frac{m}{2}v^2 + \frac{1}{2}I\omega^2$$

where  $m$  is the mass of the sphere,  $\vec{v}$  is the velocity of its center of gravity,  $\vec{\omega}$  is the spin velocity and  $I$  is the inertia. For a homogeneous sphere of radius  $r$  the inertia is defined by

$$I = \frac{2}{5}mr^2$$

From the no slipping condition ( $|r\omega| = |v|$ ) we obtain in cartesian coordinates:

$$T = \frac{7}{5} \frac{m}{2}v^2 = \frac{7}{5} \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Figure 3 shows a cross-section of the tilted cone where, in the local coordinte system,  $x$  points to the left,  $y$  points out of the page and  $z$  points upwards. If we define the tilt of the wheel along the  $y$ -axis to be zero then the gravitational force field can be expressed in local coordinates as

$$\mathbf{F} = (mg \sin \varepsilon, 0, -mg \cos \varepsilon)$$

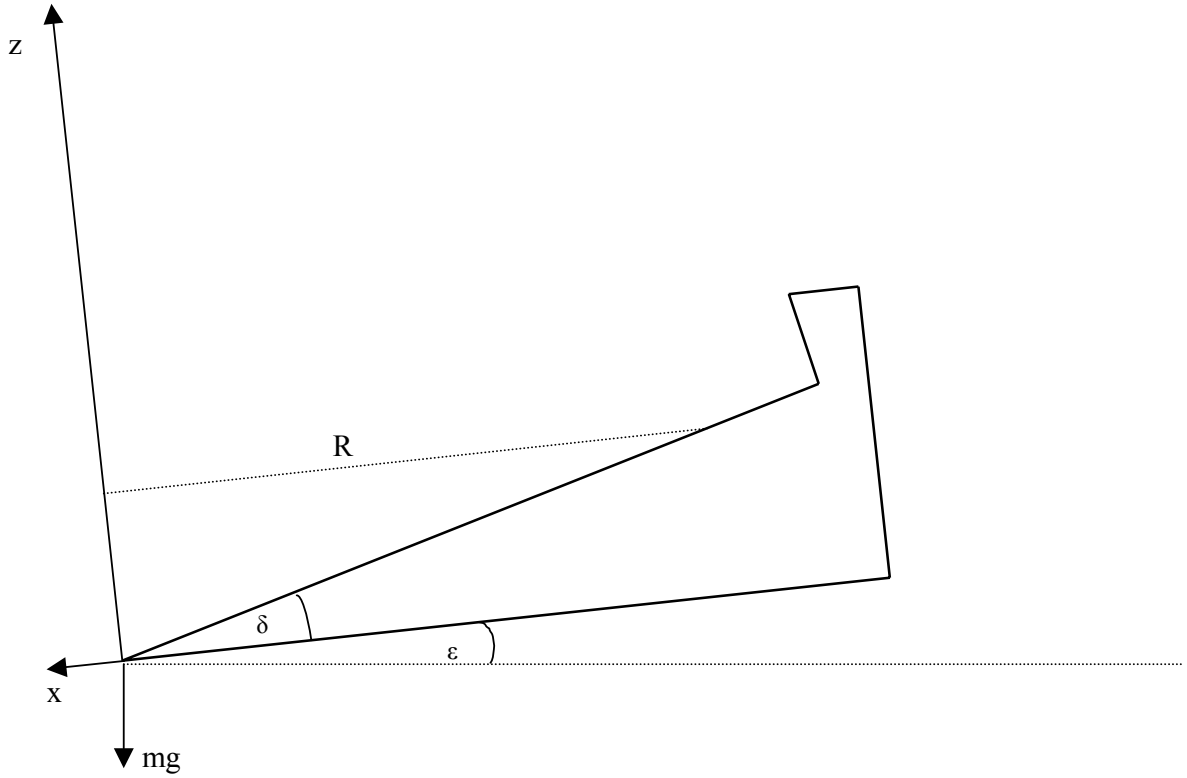
So the potential energy  $V = V(x, y, z)$  is given by:

$$V = mg(\sin \varepsilon \cdot x - \cos \varepsilon \cdot z)$$

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<sup>4</sup>The function  $S$  has actually two minima in the interval  $[0, 2\pi]$  with identical values. We always choose the physically meaningful solution with  $\eta > 0$ .

<sup>5</sup>You 'see' the ball coming and can compensate your timing reaction accordingly and in your head you can 'interpolate backwards' to identify the fall-sector.

Figure 3: *cross-section of tilted roulette wheel*

In cylindrical coordinates  $(R, \theta, z)$  defined by:

$$\begin{aligned} x &= R \cos(\theta + \varphi) \\ y &= R \sin(\theta + \varphi) \\ z &= z \end{aligned}$$

the  $z$ -coordinate is related to the  $R$ -coordinate through the angle  $\delta$  of the cone:

$$z = R \tan \delta$$

so that we can write for the kinetic and potential energy:

$$\begin{aligned} T &= \frac{7}{5} \frac{m}{2} (\dot{R}^2 / \cos^2 \delta + R^2 \dot{\theta}^2) \\ V &= mgR(\sin \varepsilon \cos(\theta + \varphi) - \cos \varepsilon \tan \delta) \end{aligned}$$

The equations of motion are obtained by defining

$$L = T - V$$

and applying the formalism of Lagrange for  $L = L(R, \dot{R}, \theta, \dot{\theta})$ :

$$\begin{aligned} \frac{d}{dt} \frac{\partial L}{\partial \dot{R}} - \frac{\partial L}{\partial R} &= 0 \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= 0 \end{aligned}$$

The variational principle of Lagrange contains all of Newtonian mechanics which in this case are exemplified by the principles of conservation of energy and conservation of angular momentum:

$$\begin{aligned}\ddot{R}/\cos\delta &= R\dot{\theta}^2\cos\delta - \frac{5}{7}g[\sin\delta\cos\varepsilon - \cos\delta\sin\varepsilon\cos(\theta+\varphi)] \\ R\ddot{\theta} &= -2\dot{R}\dot{\theta} - \frac{5}{7}g\sin\varepsilon\sin(\theta+\varphi)\end{aligned}$$

The above equations define the motion in a conservative system without dissipative forces. If we include air resistance and surface friction we obtain:

$$\begin{aligned}\ddot{R} &= -\tilde{\alpha}|v|\dot{R} - \tilde{\beta}_s\frac{|v|}{R}\dot{R} - \tilde{\beta}_c\frac{g}{|v|}\dot{R} + \left[R\dot{\theta}^2 - \frac{5}{7}g(\tan\delta\cos\varepsilon - \sin\varepsilon\cos(\theta+\varphi))\right]\cos^2\delta \\ \ddot{\theta} &= -\tilde{\alpha}|v|\dot{\theta} - \tilde{\beta}_s\frac{|v|}{R}\dot{\theta} - \tilde{\beta}_c\frac{g}{|v|}\dot{\theta} - 2\dot{R}\dot{\theta}/R - \frac{5}{7}\frac{g}{R}\sin\varepsilon\sin(\theta+\varphi)\end{aligned}\quad (44)$$

with  $|v| = \sqrt{\dot{R}^2/\cos^2\delta + R^2\dot{\theta}^2}$ .

For  $R = R_0 = \text{const}$ , implying  $\ddot{R} = \dot{R} = 0$ , the above equations reduce respectively to the exit condition (27) and the equations of motion on the rim (1), as they should<sup>6</sup>.

The conditions of friction and air resistance when the ball is rolling inside the bowl are different from those when the ball is on the rim. However, if we make some assumptions we can relate the identified parameters  $\alpha, \beta$  to the coefficients of wind friction and rolling friction in equations (44). Setting 'ex cathedra'  $\mu_0 = 0.001$  we obtain  $\mu_1$  from:

$$\mu_1 = \left(\frac{7}{5}\frac{R_0}{g}\beta + 0.001\cos\delta\right)/\sin\delta$$

We then set:

$$\begin{aligned}\tilde{\alpha} &= [\alpha - (\mu_1\cos\delta + 0.001\sin\delta)]/R_0 \\ \tilde{\beta}_s &= f_f \cdot \mu_1 \sin\delta \\ \tilde{\beta}_c &= f_f \cdot \frac{5}{7}\mu_1 \cos\delta\end{aligned}$$

Here,  $R_0$  designates the ball track radius and  $f_f$  is a fudge factor in the range  $[0, 1]$ . The fudge factor provides a "handle" to fine-tune the predictions in the final phase before the ball hits an obstacle and is scattered into the rotor. From equations (44) it follows that the slowing-down in the  $R$ -direction is mainly due to the pull of gravity when the ball loses angular velocity  $R\dot{\theta}$  (decreasing the centrifugal force which is acting in the opposite direction) and that the friction term is the main factor influencing the slowing-down in the  $\theta$ -direction. Air resistance only plays a minor role as the ball velocity has already decayed to below 0.5 rps at this point<sup>7</sup>.

<sup>6</sup>except for the fact that the friction term for the ball pressing against the rim is not present

<sup>7</sup>There is also a short transitional phase (which is not covered by our model) when the ball separates from the rim and then has to adjust its spin to the new geometric situation. Thus the ball will experience a small acceleration and depending on the magnitude of the tilt and the stator inclination angle it can even 'reconnect' to the rim after having separated from it.



With the 'initial conditions' at  $t = 0$  (when the ball comes off the rim) defined by

$$\begin{aligned} R(0) &= R_0, \quad \dot{R}(0) = 0 \\ \theta(0) &= \theta_f, \quad \dot{\theta}(0) = \Omega_f \end{aligned}$$

equations (44) are integrated numerically (using e.g. the computer codes STEP/INTRP, ROOT documented in [5]) until a suitable stopping condition (such as the collision with a vertical or horizontal deflector) is detected. In this way, a collision time  $t_c$  and a collision distance  $\theta_c$  is obtained for each exit angle  $\theta_f$  and corresponding exit velocity  $\Omega_f$  (according to the number of sectors defined). Once the values of  $t_c$  and  $\theta_c$  have been identified they remain constant for a particular wheel/ball configuration and tilt condition.

## 7 Summary

The "algorithms" outlined in the sections 1 to 5 constitute a real-time approach for determining the fall-position and fall-time of a roulette ball spinning on a tilted roulette wheel. Section 6 describes the path of the ball from where it leaves the rim and spirals down the bowl of the wheel until it collides with one of the metal deflectors. To the extent that the models of section 2 and section 6 coincide with reality the described method will accurately predict the balls motion before colliding with one of the deflectors and can be incorporated into an overall procedure for physically predicting roulette.

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## APPENDIX

## Rotor Dynamics

To accurately predict the outcome of a game of roulette we need to know the position of the rotating wheel head (usually the green zero on the rotor) at every instant in time during play. Newtons second law for a pure rotational motion can be stated as:

$$I\dot{\omega} = \tau$$

where  $I$  is the inertia of the rotor,  $\omega$  its angular velocity and  $\tau$  is the externally applied torque. If we approximate the rotor shape by the shape of a solid disc we can write for the inertia:

$$I = \frac{M}{2}R^2$$

where  $M$  and  $R$  is the mass and the radius of the rotor, respectively.

The rotor rotates on a metal shaft and is usually supported by two ball bearings. In this configuration, the friction torque  $\tau$  is given by:

$$\tau = -\mu Mg(r_1 + r_2)$$

where  $\mu = 0.001$  is the coefficient of rolling friction and  $r_1, r_2$  are the race track radii of the two ball bearings. The coefficient of rolling friction for ball bearings is a constant at low rotational frequencies (e.g. 0.33rps). This leads to the following equation of motion<sup>8</sup>:

$$\frac{M}{2}R^2\dot{\omega} = -\mu Mg(r_1 + r_2)$$

or, in terms of frequency  $\nu = \omega/2\pi$ :

$$\dot{\nu} = -k, \quad \left[ k = \mu \cdot \frac{g}{\pi} \cdot \frac{r_1 + r_2}{R^2} \right]$$

We then have for the rotor frequency and rotor position:

$$\begin{aligned} \nu(t) &= \nu_0 - k \cdot t \\ x(t) &= \nu_0 \cdot t - \frac{k}{2} \cdot t^2 \end{aligned}$$

with  $\nu(0) = \nu_0$  and  $x(0) = 0$ .

When the time  $T_0$  for one revolution is known,  $\nu_0$  is determined by:

$$\nu_0 = \frac{1 + \frac{k}{2}T_0^2}{T_0}$$

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<sup>8</sup>we can neglect air resistance because of the large mass, the low velocity and the streamlined aerodynamic shape of the rotor

Identification of the rotor deceleration parameter  $k$  is done by measuring the revolution times  $t_i$  when the wheel head consecutively passes a fixed reference point on the stator and then minimizing:

$$S(k, \nu_0) = \sum_{i=1}^n \left\{ i - \left[ \nu_0 \cdot t_i - \frac{k}{2} \cdot t_i^2 \right] \right\}^2$$

or (by straight-line fit):

$$S(k, \nu_0) = \sum_{i=1}^n p_i \left\{ \frac{i}{t_i} - \left[ \nu_0 - \frac{k}{2} \cdot t_i \right] \right\}^2$$

with weights  $p_i = t_i^2$ .

On our 60cm (professional) roulette wheel the rotor deceleration constant  $k$  was found to be approximately 0.006, so that the rotor decelerates from  $\nu_0=0.33$ rps to  $\nu_1=0.2$ rps in 20 seconds.